WSF Parameterization-Based Multiuser CFO and DOA Estimation for Frequency-Interleaved OFDMA/SDMA Uplink

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Abstract

This letter presents a joint estimation approach for carrier frequency offset (CFO) and direction of arrival (DOA) based on weighted subspace fitting (WSF) parameterization technology, tailored for interleaved OFDMA/SDMA uplink systems. The multidimensional WSF cost function associated with CFO and DOA is reformulated into two independently parameterized subspaces, enabling separate estimation of each parameter. Compared with conventional WSF algorithms, the proposed method offers reduced computational complexity and eliminates the need for pair matching. Extensive simulation results are provided to demonstrate the effectiveness and robustness of the proposed approach.

Keywords: Carrier frequency offset, Direction of arrival, Weighted subspace fitting, Interleaved OFDMA/SDMA.

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基於 WSF 參數化的多用戶同步與空間參數估計於交錯式 OFDMA/SDMA 上行系統

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摘要

本研究提出了一種基於加權子空間擬合（WSF）參數化技術的載波頻率偏移（CFO）與到達方位角（DOA）聯合估計方法，專為交錯式 OFDMA/SDMA 上行鏈路系統設計。該方法將與 CFO 和 DOA 相關的多維 WSF 代價函數重新構建為兩個可獨立參數化的子空間，從而實現對各參數的分離估計。與傳統 WSF 演算法相比，所提出的方法不僅具有較低的計算複雜度，還免除了配對匹配的需求。最終模擬與比較結果證明了本研究之方法有效性與穩健性。

關鍵字：載波頻率偏移、到達方位角、 權重子空間調整、交錯式 OFDMA/SDMA

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1. Introduction

In recent years, several technologies have drawn a lot of attention, such as interleaved orthogonal frequency division multiple access/space division multiple access (OFDMA/SDMA) and multiple-input multiple-output (MIMO). They are discussed to provide high data rate and high quality of service to satisfy users transmitting large amount of multimedia (Gong et al., 2021). In OFDMA systems, the carrier frequency offset (CFO) induced by oscillator mismatch and/or Doppler shift will destroy the orthogonality among different subcarriers and incur inter-carrier interference and multiple access interference, which seriously degrade the system performance. Besides, MIMO information is also critical to synchronization and channel estimation, and direction of arrival (DOA) is useful in beamforming. It is importance to estimate the CFO and DOA with high resolution and estimation accuracy for interleaved OFDMA/SDMA uplink systems.

In order to deal with this problem, it is shown in Yao et al.(2017) that the weighted subspace fitting (WSF) method always outperforms the deterministic maximum likelihood estimator for the estimation problem. However, the WSF estimator is still subject to high computational cost, partially attributable to the estimation of the signal subspace. A hierarchical estimation of signal parameters via rotational invariance techniques algorithm is proposed in Wu et al.(2010) for joint CFO and DOA estimation, which reduces computational complexity and no peak ambiguity problem. CFO and DOA are estimated in an hierarchical tree structure where two CFO estimations, and one DOA estimation are employed alternatively. In addition, in this hierarchical structure, these CFO and DOA parameters are interdependent. Therefore, it needs an additional pairing procedure to handle these independent parameters. In this letter, we propose a joint CFO and DOA estimation method for interleaved OFDMA/SDMA uplink systems with improving computation efficient for conventional WSF estimator. The main contributions of this letter can be summarized as follows: 1) We transform CFO and DOA related multiple dimensions WSF function into two independent parameterized multiple dimensions. 2) The proposed method directly estimates CFO and DOA and also is automatically pairing.

2. Signal Model

Consider the interleaved uplink of an OFDMA/SDMA system with N subcarriers in which K active users simultaneously communicate with the base station (BS) through an independent multipath channel, where the BS is equipped with M uniformly spaced antennas. Assume that there are N subcarriers which are divided into Q $(Q>M)$ subchannels with each subchannel having $P=N/Q$ subcarriers and the subcarriers assigned to different users are interleaved over the whole bandwidth. And, each subcarrier is exclusively used by only one user. subchannel q $(q=0,1\cdots$ contains the subcarriers with index $\{q, Q+q, \cdots$ $)Q+q\}$. For the sake of convenience, we assume that the coarse time and frequency synchronization have been completed, and the fractional time offset and CFO are considered. After passing through the multipath channel and removing the cyclic prefix, the received signal from the kth user is $r_k(n)=\sum_{p=0}^{P-1}X_k(p)H_k(p)\ e^{j2\pi np/P}e^{j2\pi n\omega_k/P}$, where $0\le n\le N-1$, $\omega_k=(q_k+\varepsilon_k)/Q$ is the effective CFO of the kth user and $\varepsilon_k\in(-0.5,\ 0.5)$ denotes the kth user's CFO normalized by subcarrier spacing $2\pi/N$. $X_k(p)$ is a set of P data streams of the kth user and $H_k(p)$ is the channel frequency response. At the BS, the received signal is $y(n)=\sum_{k=1}^{K} r_k(n)+z(n)$ at the reference antenna element, where $z(n)$ is the additive white Gaussian noise with zero mean and variance σ_n^2. It is noted that the received signal set has a special periodic feature with every P samples (Chang & Shen, 2015), i.e., $r_k(n+vP)=e^{j2\pi v\omega_k} r_k(n)$, where v $(0 \le v \le Q-1)$ is an integer. The received one OFDMA block of the kth user at the mth antenna of the BS can be stacked into a $Q\times P$ matrix and can be expressed as

$$\bar{\mathbf{Y}}_k^m=\mathbf{b}(\omega_k)a_m(\theta_k)[\mathbf{d}_k^T \quad (1)$$

where and $(\bullet)^T$ are represented as Hadamard product and transpose operation, respectively. $a_m(\theta_k)=e^{-j2\pi\Delta(m-1)\sin\theta_k/\lambda}$, $\mathbf{b}(\omega_k)=[1,\ e^{j2\pi\omega_k},\ \cdots\ \ -1)\omega_k]^T$ is the CFO vector, $\mathbf{d}_k=[1,\ e^{j2\pi\omega_k/P},\ \cdots\ \ -1)\omega_k/P]^T$, and $\mathbf{s}_k=[X_k(0)H_k(0),\ \ X_k(1)H_k(1),\ \cdots\ \ -1)H_k(P-1)]^T$. λ is the wavelength of the impinging signal, $\Delta=0.5\lambda$ is the distance between two adjacent antennas, and θ_k is the incident angle of the kth user. The signal received by the mth antenna is denoted as

$$\sum_{k=1}^{K} \ {}^{m}_{k}+\bar{\mathbf{Z}}^m \quad (2)$$

where $\bar{\mathbf{Z}}^m$ is an $Q\times P$ noise matrix for $m=1,\ 2,\ \cdots$. We first rearrange the data samples as an $QM\times P$ matrix $\mathbf{Y}=[\ ,\ ,\ \cdots$ given by

$$\mathbf{Y}=\mathbf{C}(\omega,\theta)\mathbf{S}_f+\mathbf{Z} \quad (3)$$

where $\mathbf{C}(\omega,\theta)=[\mathbf{c}(\omega_1,\theta_1),\ \mathbf{c}(\omega_2,\theta_2),\ \cdots\ \ \theta_K)]$ is the $QM\times K$ matrix with $\mathbf{c}(\omega_k,\theta_k)=\mathbf{a}(\theta_k)\otimes\mathbf{b}(\omega_k),\ k=1,\ 2,\ \cdots$. $\mathbf{a}(\theta_k)=[1,\ e^{-j2\pi\Delta\sin\theta_k/\lambda},\ \cdots\ \ M-1)\sin\theta_k/\lambda]^T$ is the $M\times 1$ steering vector, $\mathbf{S}_f=\mathbf{D}$ with $\mathbf{D}=[\mathbf{d}_1,\ \mathbf{d}_2,\ \cdots$ and $\mathbf{S}=[\mathbf{s}_1,\ \mathbf{s}_2,\ \cdots$, and $\mathbf{Z}=[\bar{\mathbf{Z}}^1,\ \bar{\mathbf{Z}}^2,\ \cdots$. The autocorrelation matrix of $\mathbf{Y}$ is $\mathbf{R}=E\{\mathbf{Y}\mathbf{Y}^H\}=$ $\mathbf{C}(\omega,\theta)E\{\mathbf{S}_f\mathbf{S}_f^H\}\mathbf{C}^H(\omega,\theta)+\sigma_n^2\mathbf{I}_{QM}$, where $\mathbf{I}_{QM}$ is the $QM\times QM$ identity matrix, $\otimes$ represents the Kronecker product, $E\{\bullet\}$ is expectation operator, and $(\bullet)^H$ is conjugate transpose. Then, the $QM\times QM$ estimate autocorrelation matrix $\hat{\mathbf{R}}$ under U data blocks is given by

$$\hat{\mathbf{R}}=\frac{1}{UP}\sum_{u=1}^{U}\mathbf{Y}(u)\mathbf{Y}^H(u) \quad (4)$$

3. CFO and DOA Estimators

3.1 Weighted Subspace Fitting Estimator

Using eigenvalue decomposition (EVD), $\hat{\mathbf{R}}$ is denoted by

$$\hat{\mathbf{R}}=\sum_{i=1}^{QM}\hat{\gamma}_i\hat{\mathbf{e}}_i\hat{\mathbf{e}}_i^H=\hat{\mathbf{E}}_s\hat{\boldsymbol{\Sigma}}_s\hat{\mathbf{E}}_s^H+\hat{\mathbf{E}}_n\hat{\boldsymbol{\Sigma}}_n\hat{\mathbf{E}}_n^H \quad (5)$$

where $\hat{\gamma}_1\ge\hat{\gamma}_2\ge\cdots$ $\cdots$ are the eigenvalues of $\hat{\mathbf{R}}$ in descending order. $\hat{\mathbf{e}}_i$ are the corresponding orthonormal eigenvector associated with $\hat{\gamma}_i$ for $i=1,\ 2,\ \cdots$.

Moreover, both $\hat{\mathbf{E}}_s = [\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2, \cdots$ and $\hat{\mathbf{E}}_n = [\hat{\mathbf{e}}_{K+1}, \hat{\mathbf{e}}_{K+2}, \cdots$ are orthogonal and span the signal subspace and noise subspace corresponding to $\hat{\mathbf{R}}$, respectively. $\hat{\mathbf{\Sigma}}_s = diag\{\hat{\gamma}_1, \hat{\gamma}_2, \cdots$ and $\hat{\mathbf{\Sigma}}_n = diag\{\hat{\gamma}_{K+1}, \hat{\gamma}_{K+2}, \cdots$. Then, statistically efficient estimation can be obtained by minimizing the following WSF problem

$$f(\omega,\theta) = tr[\mathbf{P}\hat{\mathbf{E}}_s \mathbf{W} \hat{\mathbf{E}}_s^H] \quad (6)$$

where $\mathbf{W} = (\hat{\mathbf{\Sigma}}_s - \hat{\sigma}_n^2 \mathbf{I}_K)^2 \hat{\mathbf{\Sigma}}_s^{-1}$ with $\hat{\sigma}_n^2 = (QM-K)^{-1} \sum_{i=K+1}^{QM} \hat{\gamma}_i$, $\mathbf{I}_K$ is the $K \times K$ identity matrix, and $(\bullet)^{-1}$ is the inversion operator. By searching $\omega \in [(-0.5/Q), (0.5+Q)/Q]$ and $\theta \in [-90^\circ, 90^\circ]$, we have the smallest peak of (6) taken as the estimates of the CFO and DOA for the users. Besides, the projection matrix $\mathbf{P}$ stands for the orthogonal projector onto the null space of the matrix $\mathbf{C}^H(\omega,\theta)$. Then, the projection matrix $\mathbf{P}$ is defined as

$$\mathbf{P} = \mathbf{I}_{QM} - \mathbf{C}(\omega,\theta)[\mathbf{C}^H(\omega,\theta)\mathbf{C}(\omega,\theta)]^{-1}\mathbf{C}^H(\omega,\theta) \quad (7)$$

The difficulty relies on how to parameterize the projector $\mathbf{P}$ because ω and θ are coupling.

3.2 Parameterization WSF Estimator

This subsection introduces the parameterization WSF estimator. First, the projection matrix $\mathbf{P}$ can be parameterized by the following two matrices $\mathbf{G}_p \in$ and $\mathbf{G}_q \in$. It is observed that $rank\{\mathbf{G}_p\} = M-K$, $rank\{\mathbf{G}_q\} = Q-K$, and $\mathbf{G}_p^H \mathbf{A}(\theta) = \mathbf{G}_q^H \mathbf{B}(\omega) = \mathbf{0}$. Defining $\mathbf{A}(\theta) = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \cdots]$ and $\mathbf{B}(\omega) = [\mathbf{b}(\omega_1), \mathbf{b}(\omega_2), \cdots \mathbf{b}(\omega_K)]$. Based on the above relationship, we can observe the following description. Let the columns of $\mathbf{G}_p$ and $\mathbf{G}_q$ span the null space of $\mathbf{A}^H(\theta)$ and $\mathbf{B}^H(\omega)$, respectively, and if defining $\mathbf{G}_1 = \mathbf{I}_M \otimes \mathbf{G}_q$ and $\mathbf{G}_2 = \mathbf{G}_p \otimes \mathbf{I}_Q$, where $\mathbf{I}_M$ and $\mathbf{I}_Q$ denote the $M \times M$ identity matrix and $Q \times Q$ identity matrix, respectively. Then $span\{\mathbf{G}_1\} \subset span\{\hat{\mathbf{E}}_n\}$, $span\{\mathbf{G}_2\} \subset span\{\hat{\mathbf{E}}_n\}$. It is shown in $\mathbf{G}_p^H \mathbf{A}(\theta) = \mathbf{G}_q^H \mathbf{B}(\omega) = \mathbf{0}$ that the columns of $\mathbf{G}_1$ and $\mathbf{G}_2$ span the column spaces, respectively. Hence, it can guarantee the following properties $\mathbf{G}_1^H \mathbf{C}(\omega,\theta) = \mathbf{A}$ $= \mathbf{0}$ and $\mathbf{G}_2^H \mathbf{C}(\omega,\theta) = (\mathbf{G}_p^H \mathbf{A})$, where stands for the Khatri Rao product. On the other hand, if considering the rank of matrix $\mathbf{G}_1$ and matrix $\mathbf{G}_2$, they have $rank(\mathbf{G}_1) = rank(\mathbf{I}_M) \times rank(\mathbf{G}_q) = M(Q-K)$ and $rank(\mathbf{G}_2) = rank(\mathbf{G}_p) \times rank(\mathbf{I}_Q) = (M-K)Q$. Herein we utilize the property of the rank of two matrices' Kronecker product (Brewer, 1978). As we know, the dimension of the noise subspace is $rank(\hat{\mathbf{E}}_n) = QM-K$. Obviously, $rank(\hat{\mathbf{E}}_n) > rank(\mathbf{G}_1)$ if $M \geq 2$ and $rank(\hat{\mathbf{E}}_n) > rank(\mathbf{G}_2)$ if $Q \geq 2$.

Such result directly indicates a fact that the column space spanned by the columns of $\mathbf{G}_1$ and the one spanned by $\mathbf{G}_2$ are all included in the noise subspace spanned by $\hat{\mathbf{E}}_n$. According to the above description, we can construct two projection matrices, $\mathbf{P}_{G_1} = \mathbf{G}_1(\mathbf{G}_1^H \mathbf{G}_1)^{-1} \mathbf{G}_1^H = \mathbf{I}_M \otimes \mathbf{P}_{G_q}$ and $\mathbf{P}_{G_2} = \mathbf{G}_2(\mathbf{G}_2^H \mathbf{G}_2)^{-1} \mathbf{G}_2^H = \mathbf{P}_{G_p} \otimes \mathbf{I}_Q$, where $\mathbf{P}_{G_p} = \mathbf{G}_p(\mathbf{G}_p^H \mathbf{G}_p)^{-1} \mathbf{G}_p^H$ and $\mathbf{P}_{G_q} = \mathbf{G}_q(\mathbf{G}_q^H \mathbf{G}_q)^{-1} \mathbf{G}_q^H$ so that one can substitute $\mathbf{P}_{G_1}$ or $\mathbf{P}_{G_2}$ for $\mathbf{P}$ in (7). Consequently, two new objective functions, which further need to be minimized separately, are reformulated as

$f(\omega) = tr[\mathbf{P}_{G_1}\hat{\mathbf{E}}_s\mathbf{W}\hat{\mathbf{E}}_s^H]$ and $f(\theta) = tr[\mathbf{P}_{G_2}\hat{\mathbf{E}}_s\mathbf{W}\hat{\mathbf{E}}_s^H]$ (Yao & Meng et al., 2017). If comparing the original objective function (6), $f(\omega,\theta)$, with functions $f(\omega)$ and $f(\theta)$, we can see that the multi-dimensional objective function is divided into two different independent multi-dimensional functions, and consequently, the computational complexity decreases. It is worth noting that the parameterization WSF requires pair matching (Yao & Meng et al., 2017) .

3.3 Signal Subspace Parameterization Estimator

In this subsection, we present the proposed signal subspace parameterization (SSP) estimator. In order to implement the minimization in $f(\omega)$ more convenient, we define

$$\mathbf{E} = \hat{\mathbf{E}}_s\mathbf{W}\hat{\mathbf{E}}_s^H = \begin{bmatrix} \mathbf{E}_{1,1} & \mathbf{E}_{1,2} & \cdots & \mathbf{E}_{1,M} \\ \mathbf{E}_{2,1} & \mathbf{E}_{2,2} & \cdots & \mathbf{E}_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{E}_{M,1} & \mathbf{E}_{M,2} & \cdots & \mathbf{E}_{M,M} \end{bmatrix} \tag{8}$$

where $\mathbf{E}_{l,h}$ is a $Q \times Q$ matrix, $l, h = 1, 2, \cdots, M$. We further take advantage of $\mathbf{E}$, only the diagonal block-matrices $\{\mathbf{E}_{m,m}\}_{m=1}^{M}$ are utilized. Thus, let $\tilde{\mathbf{E}} = \sum_{m=1}^{M}\mathbf{E}_{m,m}$ and we can get

$$\hat{\omega} = \min_{\omega}\ tr\{\mathbf{P}_{G_q}\tilde{\mathbf{E}}\}, \tag{9}$$

By searching ω, the smallest peak of (9) taken as the estimate of the CFO for the users.

Next, the DOA estimation can be achieved by the following optimization problems (Zhang et al., 2010)

$$\min_{\theta}\ \mathbf{a}^H(\theta)\bar{\mathbf{E}}(\hat{\omega})\mathbf{a}(\theta),\ \text{s.t. } \mathbf{e}_1^T\mathbf{a}(\theta) = 1 \tag{10}$$

where $\bar{\mathbf{E}}(\hat{\omega}) = Q\mathbf{I}_M - [\mathbf{I}_M \otimes \mathbf{b}(\hat{\omega})]^H\hat{\mathbf{E}}_s\hat{\mathbf{E}}_s^H[\mathbf{I}_M \otimes \mathbf{b}(\hat{\omega})]$ and $\mathbf{e}_1 = [1,\ 0,\ \cdots,\ 0]^T$ is the $M \times 1$ vector whose first element is one and other elements are zero. Due to we have acquired the estimated $\hat{\omega}$ information, if we take them into (10), then auto-paired steering vectors can be easily derived by means of Lagrange multiplier method, i.e.,

$$\mathbf{a}(\theta_k) = \bar{\mathbf{E}}^{-1}(\hat{\omega}_k)\mathbf{e}_1 / \mathbf{e}_1^T\bar{\mathbf{E}}^{-1}(\hat{\omega}_k)\mathbf{e}_1,\ k = 1,\ 2,\ \cdots \tag{11}$$

The θ information can be obtained from those steering vectors by means of least squares (LS) principle. The LS fitting problem is shown as follows

$$\min_{\hat{g}_k}\ \|\mathbf{p}_c\hat{g}_k - \bar{\mathbf{a}}_k\|^2 \tag{12}$$

where $\mathbf{p}_c = [0,\ -2\pi\Delta/\lambda,\ \cdots,\ -2\pi\Delta(M-1)/\lambda]^T$ and we get

$$\begin{aligned} \bar{\mathbf{a}}_k &= angle\{\mathbf{a}(\theta_k)\} \\ &= [0, -2\pi\Delta\sin\theta_k/\lambda,\ \cdots,\ -2\pi\Delta(M-1)\sin\theta_k/\lambda]^T \end{aligned} \tag{13}$$

where $angle\{\bullet\}$ is to get the phase angles for each element of complex array. The LS solution can be obtained by $\hat{g}_k = (\mathbf{p}_c^H\mathbf{p}_c)^{-1}\mathbf{p}_c^H\bar{\mathbf{a}}_k$. Finally, the CFO and DOA are estimated via

$$\hat{\varepsilon}_k = \hat{\omega}_k Q - q_k,\ k = 1,\ 2,\ \cdots \tag{14}$$

$$\hat{\theta}_k = \sin^{-1}(\hat{g}_k),\ k = 1,\ 2,\ \cdots \tag{15}$$

In summary, the detailed steps of the proposed SSP estimator are summarized as follows:

Step 1. Calculate the autocorrelation matrix of $\hat{\mathbf{R}}$ by (4).

Step 2. Perform the EVD of $\hat{\mathbf{R}}$ to obtain the signal subspace $\hat{\mathbf{E}}_s$ and matrix $\mathbf{W}$.

Step 3. Construct matrix $\tilde{\mathbf{L}}$ and perform (9) to obtain $\hat{\omega}_k,\ k = 1,2,\cdots$.

Step 4. Substituting $\mathbf{E}_s\mathbf{E}_s^H$ and $\hat{\omega}_k$ into (10) and using (12) to obtain $\hat{g}_k,\ k = 1,2,\cdots$.

Step 5. Convert $(\hat{\omega}_k,\ \hat{g}_k)$ to $(\hat{\varepsilon}_k,\ \hat{\theta}_k)$ using (14) and (15).

4. Simulation Results

This section provides simulation results to evaluate the performance of the proposed method. Consider a uniform linear array with half-wavelength spacing at BS. Assume that the number of antennas and the number of users on the BS are and, respectively. The total number of subcarriers in the OFDMA/SDMA system is, which is divided into subchannels and each user is allocated to subcarriers. Let and represent the CFO and DOA values for the eight users, respectively. The transmitted symbols are binary phase shift keying modulation. In simulation, we also assume that channel state will not vary in the duration of one OFDMA block. The root mean square (RMSE) of CFO and DOA are defined as and with Monte Carlo trials, respectively. Three estimators, including the WSF (Yao & Zhang et al., 2017), parameterization WSF (Yao & Meng et al., 2017), and the proposed SSP estimators, are conducted for comparison in terms of the RMSE of the CFO and DOA estimates.

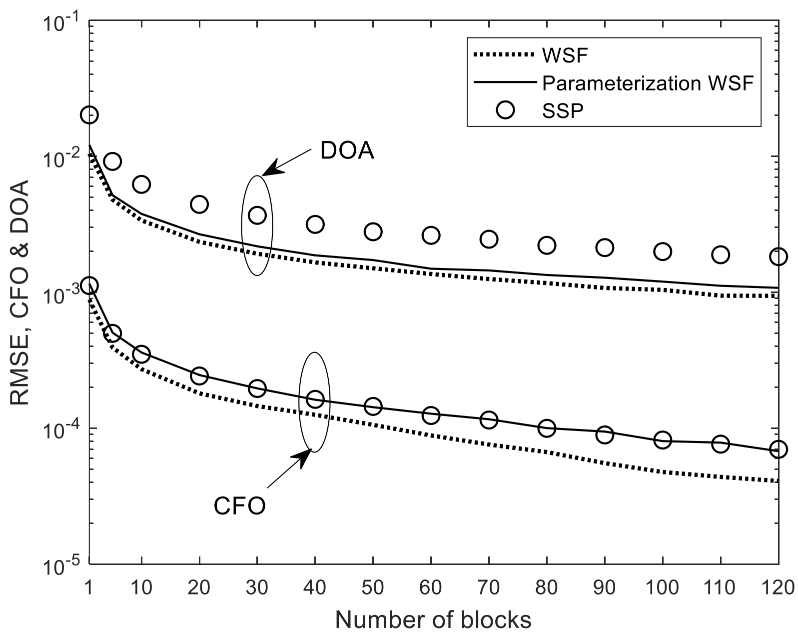

Figure 1. *RMSE of CFO and DOA estimation versus number of blocks*.

Figure 1 shows RMSE of CFO and DOA versus the number of blocks under $\mathrm{SNR} = 10\mathrm{dB}$ and Figure 2 shows RMSE of CFO and DOA versus SNR under one OFDMA/SDMA data block. In the CFO estimate, we can see that the proposed SSP has the approximate performance as the parameterization WSF in all data blocks. In addition, the performance of all estimators improves when the number of data blocks increases. On the other hand, compared with the parameterization WSF, the performance of the SSP in DOA estimation is slightly worse due to the use of fewer degrees of freedom, but the RMSE of its DOA estimation is still acceptable. In particular, the proposed SSP provides relatively low computational load in DOA estimation compared to the WSF and parameterization WSF.

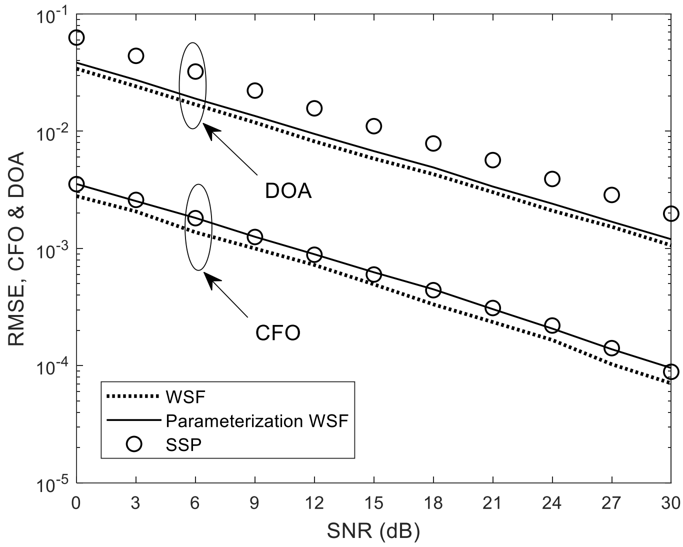

Figure 2. *RMSE of CFO and DOA estimation versus SNR.*

Table 1. *Computational complexity analysis*

Estimators	Computational complexities
WSF	$12(QM)^3+(QM)^2P+\{F_\omega F_\theta[2K^3+3(QM)^2K+3QMK^2]\}^K$
Parameterization WSF	$12(QM)^3+(QM)^2P$ $+\{F_\omega[2(QM)^2K+QMK^2+2(Q-K)^3+2(Q-K)^2Q+(Q-K)Q^2]\}^K$ $+\{F_\theta[2(QM)^2K+QMK^2+2(M-K)^3+2(M-K)^2M+(M-K)M^2]\}^K$
SSP	$12(QM)^3+(QM)^2P+(QM)^2K+QMK^2+K[3M+2]$ $+\{F_\omega[2(Q-K)^3+2(Q-K)^2Q+(Q-K)Q^2+Q^3]\}^K$

Final, we analyze the computational complexity of complexity multiplications (CM) for all estimators. Assuming K users, M antenna elements, Q subchannels, and P subcarriers, for each test, the computational complexities of calculating $\hat{\mathbf{R}}$ and EVD for an $QM\times QM$ correlation matrix require approximately $(QM)^2\times P$ and $12(QM)^3$ CM (Golub & Van, 1996), respectively. Due to the parameterization WSF and WSF involves user searching in K-dimension, hence the computation required by WSF is $\{F_\omega F_\theta[2K^3+3(QM)^2K+3QMK^2]\}^K$ CM; the parameterization WSF requires $\{F_\omega[2(QM)^2K+QMK^2+2(Q-K)^3+2(Q-K)^2Q+(Q-K)Q^2]\}^K$

and $\{F_\theta[2(QM)^2K + QMK^2 + 2(M-K)^3 + 2(M-K)^2M + (M-K)M^2]\}^K$ CM, respectively. The proposed SSP requires $\{F_\omega[2(Q-K)^3 + 2(Q-K)^2Q + (Q-K)Q^2 + Q^3]\}^K + K[3M+2]$ CM, where $K[3M+2]$ is the computational complexity of DOA pairing. In summary, the required CM are listed in Table 1.

Next, assume that the searching grids of CFO and DOA are denoted as μ_ω and μ_θ, respectively. The searching grid of CFO of the WSF, parameterization WSF, and SSP is set to $\mu_\omega = 10^{-5}$. Except for the SSP, the DOA searching grid of the WSF and parameterization WSF is set to $\mu_\theta = 10^{-3}$. Figure 3(a) shows the number of required CM in logarithmic scale versus the number of subchannels for the case of $M = 12$ antennas, whereas Figure 3(b) shows the number of required CM in logarithmic scale versus the number of antennas for $Q = 32$ subchannels. Estimating CFO and DOA require calculations of $F_\omega = [(1+\frac{1}{Q})\frac{1}{\mu_\omega}] + 1 = 103{,}126$ and $F_\theta = \frac{180}{\mu_\theta} + 1 = 180{,}001$ times, respectively. It is worth noting that the computational load of WSF is considerable. From these two figures, it shows that the proposed SSP indeed has reduced computational burden.

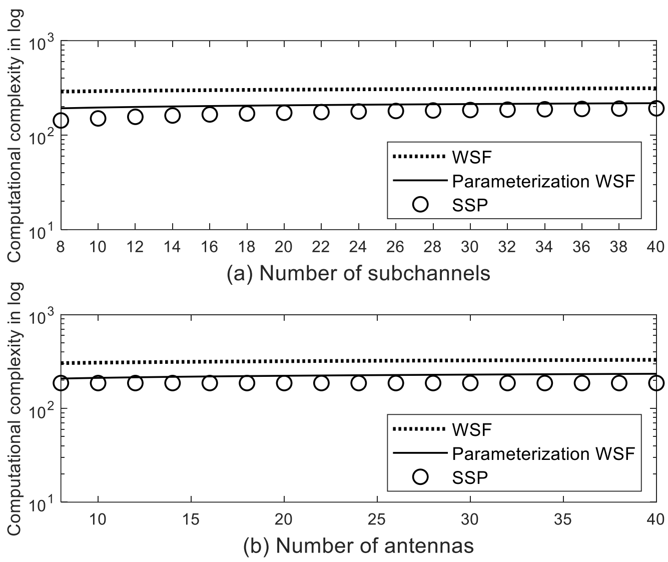

Figure 3. *Computational complexity versus (a) number of subchannels, (b) number of antennas.*

5. Conclusions

This letter has presented a joint CFO and DOA estimation method based on WSF parameterization technology for interleaved OFDMA/SDMA uplink. By designing two orthogonal projection projectors onto the null space of the joint CFO and steering vectors. Simulation results verified that the proposed estimator preserves less computational load and also does not require the pairing procedure.

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